An Empirical Testing of Capital Asset Pricing Model in Social Media Stock Market

Zihao Zhang[1,*]

[1]The University of Queensland, City, Brisbane, Australia

[*]Corresponding author e-mail: zihao_tom0713@hotmail.com

Abstract: In modern finance theory, the Capital Asset Pricing Model (CAPM) is one of the most fundamental models used to predict the price of assets. It states that the expected return of a risky asset is linearly related to its systematic risk factor, i.e., beta. In this paper, we aim to test the effectiveness of CAPM in social media stock market. The study is conducted for a period of 301 trading days ranging from April 22nd, 2019 to June 30th, 2020 with daily return data and Fama-MacBeth rolling OLS regression methodology being applied. The results show that CAPM has a certain explainability and can be helpful for investors when making decisions.

1. Introduction

The relationship between portfolio returns and market returns can be investigated using the CAPM, which becomes one of the significant benchmarks in modern finance theories. In empirical finance, the beta coefficient calculated by CAPM is widely used to measure risks.

Markowitz (1952) proposed the Mean-Variance model for portfolio investment [1]. On this basis, Sharpe (1964), Lintner (1965), and Mossin (1966) presented the Capital Asset Pricing Model (CAPM) [2][3][4]. Besides, Black (1972) performed an empirical test on the investment portfolio, verifying that the CAPM needs to be adjusted and then a zero-beta model should be proposed when there is no risk-free asset [5]. Merton (1973) constructed a continuous-time portfolio and asset pricing theory and developed the CAPM into an Intertemporal CAPM (ICAPM) [6]. Ross (1976) put forward a new asset pricing model-arbitrage pricing theory (APT) [7]. Then, the linear relationship between the balanced return of risk assets and multiple factors are obtained according to the principle of no-arbitrage. Lucas (1978) and Breeden (1979) established a consumption-based asset pricing model (CCAPM), assuming that consumers aim to maximize the current and future total utility and consumers consume and invest under budget constraints. The model combines the consumption choice theory with the CAPM theory [8][9]. FAMA and French (1992) performed the empirical research on the stock market, demonstrating that the difference between the returns of different stocks cannot be explained by the beta value of the CAPM; therefore, P/E ratio, market value, and book-to-market ratio were introduced [10]. Arrow (1952) proposed the principle of random discount for asset pricing by analysing consumer choice behaviour [11][12]. Hansen and Richard (1987) explicitly used "stochastic discount factors"[13]. Cochrane (2001) provided a Stochastic discount factor pricing model (SDF) [14]. The model established a general connection between the future payment of financial assets and the current price, revealing the general logic of asset pricing.

From another perspective, the questioning of the CAPM has never stopped. Litzenberger and Ramaswamy (1979) conducted empirical research on the New York Stock Exchange from 1936 to 1977, illustrating that the dividend yield also has a positive effect on the stock yield [15]. Jegadeesh and Titman (1993, 2001) proposed the "inertial effect"[16], which refers to the tendency of stock returns to continue the original movement direction, suggesting that the stocks with higher returns in the past period will still obtain higher returns in the future compared to the stocks with lower returns in the past.

However, whether the CAPM is effective in social media stock market or not has not been

researched yet. In this paper, we mainly focus on validating the effectiveness of CAPM in social media stock market using Fama-MacBeth technique. The following paper are organized as follows: Firstly, we briefly introduce the theoretical model of CAPM including its key assumptions. Then, eight social media stocks in American capital market are chosen for empirical analysis and validation. Based on the results, statistical methods are applied to test whether the CAPM is effective or not. Finally, some conclusions and further analysis are made.

2. Theoretical Model

Methodology

The CAPM builds upon the theory of Capital Market Theory which is an extension of Mean-Variance Portfolio Model developed by Markowitz in 1959. In this model, investors are assumed to choose portfolios that are mean-variance-efficient which is to say that the variance of portfolio return is minimal when expected return is given or the expected return of portfolio is maximal given known variance. Besides assumptions made by Markowitz, Sharpe and Linter add another two assumptions. In summary, key assumptions includes:

(1) Investors evaluate portfolios based solely on the expected returns and standard deviation.

(2) There are no transaction costs and taxes in the market, and all assets can be tradable. The market is perfectly competitive with zero information cost. All participants are rational and accept all information simultaneously.

(3) Both purchase and sale transactions can be undertaken without any limit.

(4) Investors can borrow or lend at a risk-free rate, no matter how much amount.

Based on these assumptions, Sharpe and Linter give the following equation which is considered to be the standard and simplest form:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \tag{1}$$

where

$E(R_i)$ = The expected return of asset i

β_i = Coefficient on RMRF, which represents the risk compensation factor of asset i.

R_f = Risk free rate

$E(R_m)$ = The expected return of the market portfolio

The explicit relationship between expected return and market beta is the basis of CAPM testing. It is obvious that asset returns are linearly related to their betas with no other explanatory risk factor. Also, expected return of the market portfolio exceeds risk free rate which is uncorrelated with the market. For years, time-series or cross-sectional regressions are used for empirical testing.

In 1973, Fama and MacBeth proposed a two-pass cross-sectional regression for testing CAPM which is also our main methdology in this paper. The procedure works as follows:

First, risk loading, or betas can be estimated for each asset by running time-series regression. This procedure is called first-pass regression.

$$R_{it} - R_{ft} = \alpha_i + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it}, i = 1,2,\cdots,N, t = 1,2,\cdots,T \tag{2}$$

where α_i and β_i are intercept and slope of the regression, respectively.

If written in a more generalized form, we can get the following term:

$$R_{it} = \alpha_i + \beta_i f_t + \varepsilon_{it}, i = 1,2,\cdots,N, t = 1,2,\cdots,T \tag{3}$$

where f_t is the risk factor like expected return of market portfolio, or other macro economic variables like GDP, CPI, etc..

In the second step, risk premium is estimated by running cross-sectional regression:

$$R_{it} - R_{ft} = \gamma_{0t} + \gamma_{1t}\widehat{\beta_i} + u_{it}, i = 1,2,\cdots,N, t = 1,2,\cdots,T \tag{4}$$

where $\widehat{\beta_i}$ is the estimated betas in equation (2). For each t, independent cross-sectional regression is undertaken. Thus, we can average parameters finally as their estimates:

$$\widehat{\gamma_0} = \sum_{t=1}^{T} \widehat{\gamma_{0t}} \tag{5}$$

$$\widehat{\gamma_1} = \sum_{t=1}^{T} \widehat{\gamma_{1t}} \tag{6}$$

Furthermore, standard errors of γ_0 and γ_1 can be calculated easily by the following equations:

$$\sigma^2(\gamma_0) = \frac{1}{T} \sum_{t=1}^{T} (\widehat{\gamma_{0t}} - \widehat{\gamma_0})^2 \tag{7}$$

$$\sigma^2(\gamma_1) = \frac{1}{T} \sum_{t=1}^{T} (\widehat{\gamma_{1t}} - \widehat{\gamma_1})^2 \tag{8}$$

This procedure is called second-pass regression. After two-step regressions, statistic tests can be undertaken to validate whether the parameters are statistically significant. Based on the results of T-tests, whether the CAPM is effective or not can be concluded.

3. Empirical Result

3.1 Data Description

The sample data selected in our experiment are eight typical social media companies including Facebook, Pinterest, Snapchat, Twitter, Sina, Weibo, Renn, and Momo. Daily returns are chosen from April 23nd, 2019 to June 30st, 2020. Dow Jones Index is selected as the market index since it is a value-weighted index that meets the requirements of the CAPM market portfolio structure. The yield of 1-year U.S. Treasury bond is regarded as the risk free rate.

In total, there are 301 observations of return data. In order to obtain reliable results, we use rolling OLS regression instead of ordinary OLS regression. Rolling window is set to 30, which is one month.

3.2 Experiment Procedure

Matlab is a useful high-level language and development tool for numerical calculation and simulation. With financial toolbox or other embedded functions, it is simple and quick to write programs, develop and analyse algorithms, or even build complex applications. In this paper, we mainly use *Matlab* to calculate and analyse the CAPM model.

Raw stock data is downloaded from Yahoo Finance website. Before further analysis, we need first convert stock price into return data. Here, we use log return instead of simple return.

$$R_{it} = ln \frac{P_{i,t}}{P_{i,t-1}} \tag{9}$$

where R_{it} denotes the return of the *i-th* stock in the *t-th* day, and $P_{i,t}$ refers to the closing price of the *i-th* stock in the *t-th* day. The same pre-processing is also applied to the market index data. After that, risk free rate are subtracted from the returns of stocks and market index in order for regression with equation 2.

As mentioned before, we use a rolling window to regress our time series data. Totally, there are 271×8 (301-30 =271 periods, 8 stocks) regressions in the first step. Then, average estimation of betas can be obtained. After that, cross-sectional OLS regressions are run for each dates assuming that betas are given from step 1 using equation 4. After two-pass regression, parameters estimated are analysed and tested using simple t-statistics.

For γ_0, we should test the following hypothesis and if the CAPM holds true, the value of γ_0 should not be significantly different from zero.

$$H_0: \quad \gamma_0 = 0 \quad \text{v.s.} \quad H_a: \quad \gamma_0 \neq 0 \tag{10}$$

For γ_1, the following hypothesis should be tested:

$$H_0: \quad \gamma_1 = E(R_m) - R_f \quad \text{v.s.} \quad H_a: \quad \gamma_1 \neq E(R_m) - R_f \tag{11}$$

3.3 Results

First-pass regression results are shown in Table 1, including estimated values of parameters, their

standard errors and corresponding T-statistic values.

Table 1 Results of first-pass rolling time series regression

Stock	$\overline{\widehat{\alpha}}$	SE($\widehat{\alpha} \times 10^{-3}$)	T-stat($\widehat{\alpha}$)	$\overline{\widehat{\beta}}$	SE($\widehat{\beta}$)	T-stat($\widehat{\beta}$)
Facebook	0.0012	0.1352	8.8466	0.9718	0.0119	81.5524
Pinterest	-0.0003	0.4031	-0.6556	0.8738	0.0184	47.4496
Snapchat	0.0028	0.3194	8.6969	0.9604	0.0183	52.3445
Twitter	-0.0001	0.3324	-0.4492	0.9300	0.0151	61.7724
Sina	-0.0010	0.2943	-3.2401	0.7721	0.0177	43.6017
Weibo	-0.0017	0.4044	-4.1814	1.8481	0.0185	99.9087
Renn	0.0003	0.9885	0.2625	1.8376	0.0287	64.0141
MOMO	-0.0020	0.2954	-6.7303	1.9163	0.0143	133.8298

where $\overline{\widehat{\alpha}}$ and $\overline{\widehat{\beta}}$ are calculated using the following equations:

$$\overline{\widehat{\alpha}}_i = \sum_{t=1}^{T_2} \widehat{\alpha}_{ti} \tag{12}$$

$$\overline{\widehat{\beta}}_i = \sum_{t=1}^{T_2} \widehat{\beta}_{ti} \tag{13}$$

where T2 is number of regression periods, i.e., 271.

As we can see from the results in the table, T-statistic values of alphas range from -6.7303 to 8.8466, with only three stocks, i.e., Pinterest, Twitter and Renn smaller than t critical value at 5% significance level with 270 (T2-1) degree of freedom. This proves that CAPM is not always efficient in each social media stock. Similarly, we can get some insights from the T-statistic values of betas which implies that no beta is equal to zero significantly. Economically speaking, each stock is exposed to systematic risk or market risk, no matter more or less, since the excess return is directly related to the excess return of the market.

With values of estimated betas, we can calculate average predicted excess return of each stock by the following equation:

$$\widehat{\overline{R}}_i = \overline{\widehat{\beta}}_i \cdot \overline{R_m} \tag{14}$$

where $\overline{\widehat{\beta}}_i$ is the average estimation value of betas in equation (13).

To see the relationship between average excess returns of each stock and their betas or predicted value, scatter figures are plotted in Figure 1 and 2, respectively.
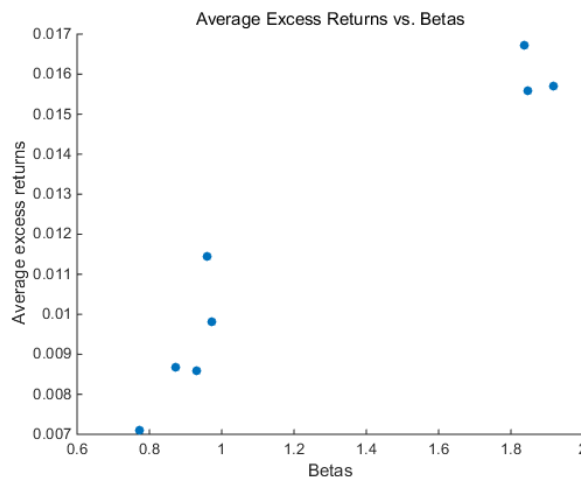

Figure 1. Average Excess Returns vs. Betas

As is shown vividly in the figures, there is no strict linear relationship between average excess

returns and betas although few points are in the straight line. This also provides conclusions that CAPM is not always efficient, which is in accordance with the results from Table 1.

As the same with Figure 1, relationship between average excess returns and predicted excess returns is shown in Figure 2.
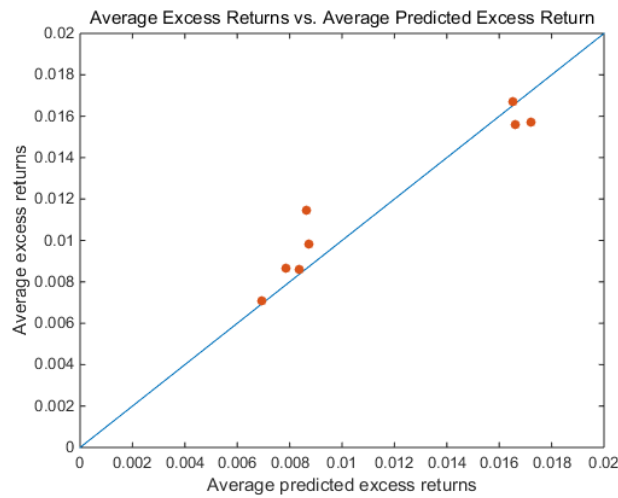

Figure 2. Average Excess Returns vs. Predicted Excess Returns

Moreover, for each stock, we can plot their time series excess return data versus predicted time series excess data which is shown in Figure 3.
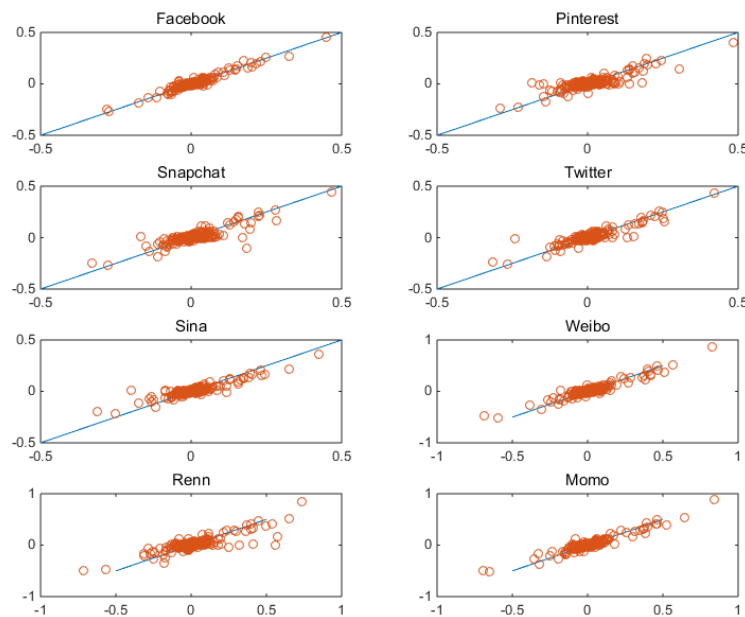

Figure 3. Excess Return of each stock vs. Predicted values

After first-pass regression, average estimation values of betas can be obtained. We then run cross sectional regression for each date period to get second-pass regression parameters which are shown in Table 2.

Table 2. Results of second-pass cross sectional regression

Parameters	Estimation	SE	T-stat	R Squared
γ_0	0.0026	0.0023	1.1013	0.3229
γ_1	0.0072	0.0044	-0.3952	

4. Discussion

As illustrated in the regression results in Table 2, we can't reject null hypothesis that $\gamma_0 = 0$ at 5% significance level because of its T-statistic value. Also, we can't reject null hypothesis that $\gamma_1 = R_m - R_f$ since its T-statistic value is smaller than 1.96 at 5% significance level. This means that the stock returns chosen in our paper are proportional to the coefficient β which proves effectiveness of the CAPM model to some extent. However, R^2 is relatively small, suggesting that the CAPM model does not fit well.

Furthermore, we can take an insight into the results of first-pass regression. From the column of T-statistic values of alpha in table 1, we can see that not all hypothesises can be rejected, reflecting that the CAPM does not always hold true. From the graphs shown in Figure 1-3, evidence that excess returns of stock and predicted values are not strictly linear can also be found.

The Fama-MacBeth method does have some advantages since it is intuitive and simple. Also, it excludes the problem of correlation of the residuals in cross-sectional regression. However, the correlation of residuals in time-series regression is not solved. Besides, the betas used in second-pass regression are estimated values obtained from first-pass regression which adopts additional errors. This is a limitation when validating the CAPM model.

5. Conclusion

In summary, CAPM is partially but not fully effective in American social media stock market according to the empirical results researched in this paper. However, there are also some drawbacks such as insufficient number of stocks, limited samples, oversimplified regression equation form, and superficial analysis of regression results. In this paper, due to limited number of social media companies in capital market, only eight stocks with 301 periods of high frequency data are chosen. Daily returns are usually more volatile compared with monthly or yearly returns which are not proper for analysing long-term trends of stock returns. Instead, we can download more original data with 10 or 20 years and convert them into monthly returns which are more suitable. Besides, more stocks should be added for thorough analysis which is more persuasive statistically.

Moreover, the practicality of the model is restricted by the strict assumptions of CAPM. Based on the results of many empirical studies, CAPM is rejected to some degree, or beta is, at least, not the only explanatory factor for stock returns. Other risk factors like size, value, momentum, etc. also have contributions to excess returns of stocks which are needed for further study.

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